Review 15 Euler's method, L'hopital's rule

Euler's method approximates y_k ← final value

given y', (x_0, y_0) {initial value problem}

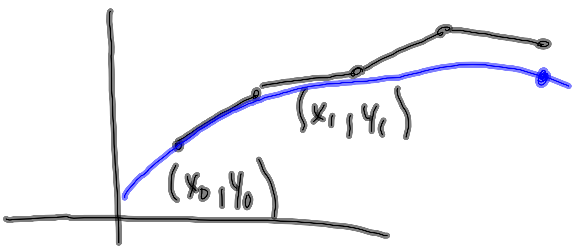
↑ initial value

$$y_{n+1} = y_n + f'(x_n, y_n)\,\Delta x$$

↖ step size

x	y	y'
x_0	y_0	.
x_1	y_1	⋮
x_2	y_2	⋮
⋮	⋮	
x_k	y_k	

Δx



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$f'(x,y) = 2x + y$ $\quad f(1) = 1$ $\quad$ (approximate) find $f(2)$

using Euler's method with steps of size .5

x	y	y'
1	1	3
1.5	2.5	5.5
2	5.25	

$1 + 3(.5)$

$2.5 + 5.5(.5)$

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l'hopital's rule   indeterminate $\frac{0}{0}, \frac{\infty}{\infty}$

If $\lim \frac{f(x)}{g(x)}$ is $\frac{0}{0}$ or $\frac{\infty}{\infty}$ indet.

then use $\lim \frac{f'(x)}{g'(x)}$

other indeterminates $\infty - \infty$, 1^∞, $0 \cdot \infty$

need fraction $\frac{0}{0}$ or $\frac{\infty}{\infty}$

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$$\lim_{x \to 0} \frac{\sin x}{x^2} = \lim_{x \to 0} \frac{\cos x}{2x} = \infty$$

$\frac{0}{0}$ $\frac{1}{0}$

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$$\lim_{x\to 0} (1+x)^{\frac{1}{x}} = ? \quad y$$

$$\lim_{x\to 0} \ln(1+x)^{\frac{1}{x}} = \ln y$$

$$\lim_{x\to 0} \frac{1}{x}\left(\ln(1+x)\right) = \ln y$$

$$\lim_{x\to 0} \frac{\ln(1+x)}{x} = \ln y$$

$$\frac{0}{0} \qquad \lim_{x\to 0} \frac{\frac{1}{1+x}\cdot 1}{1} = 1 = \ln y$$

$$e^1 = e^{\ln y}$$

$$e = y$$

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$$\lim_{x\to\infty} \left(\frac{x}{1+x}\right)^x = y$$

$$\lim_{x\to\infty} \ln\left(\frac{x}{1+x}\right)^x = \ln y$$

$$x\ln\left(\frac{x}{1+x}\right) =$$

$$\frac{0}{0} \qquad \lim_{x\to\infty} \frac{\ln\left(\frac{x}{1+x}\right)}{\frac{1}{x}} = \lim_{x\to\infty} \frac{\frac{1+x}{x}\cdot\frac{(1+x)1-x\cdot 1}{(1+x)^2}}{-\frac{1}{x^2}}$$

$$\boxed{y = \frac{1}{e}} \qquad = \lim_{x\to\infty} \frac{x^2}{-1}\cdot\frac{1+x}{x}\cdot\frac{1}{(1+x)^2}$$

$$= -1 = \ln y$$

$$e^{-1} = y$$

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